

# Stripes And Nodal Fermions As Two Sides Of The Same Coin.

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One of the central conceptual problems in High  $T_c$  superconductivity is to reconcile the abundant evidence for stripe-like physics at ‘short’ distances with the equally convincing evidence for BCS-like physics at large distance scales (the ‘nodal fermions’). Our central hypothesis is that the duality notion applies: the superconductor should be viewed as a condensate of topological excitations associated with the fully ordered stripe phase. As we will argue, the latter are not only a form of ‘straightforward’ spin and charge order but also involve a form of ‘hidden’ or ‘topological’ long range order which is also responsible for the phenomenon of spin-charge separation in 1+1D. The topological excitation associated with the destruction of this hidden order is of the most unusual kind. We suggest that the associated disorder field theory has a geometrical, gravity like structure concurrent with topological phases with no precedent elsewhere.

## THE PARADOX

Paradoxes are among the best weaponry available to a scientist. The paradox in science is associated with a flaw in the theoretical understanding on the most basic level. Recently an interview with Edward Witten was broadcasted on dutch TV. The interviewer tried to corner Witten, arguing that quantum-gravity is a shaky affair because it is not accessible by experimental means. Witten was prepared for this question, arguing that the situation is not that bad because quantum gravity is firmly rooted in a grand paradox. Einstein’s theory of gravity and quantum mechanics are fundamentally incompatible. This is intimately linked to basic assumptions that are so self evident that they are not even explicitly formulated. The pursuit of string-theory should be considered as an attempt to lay bare these hidden assumptions, and in this sense progress is made.

We want to suggest that high  $T_c$  superconductivity is in a similar state. The field revolves around a grand paradox, with the added merit that it has experimental physics on its side.

The arrival of a paradox is accompanied by raging controversy, and the controversy in high  $T_c$  superconductivity is not easy to overlook. The community has bifurcated in two schools of thought. The first school rests on the conceptual backbone of conventional BCS theory and has been quite successful in addressing the physics of the fully developed superconducting state[1]. Their stronghold are the quasiparticles associated with the d-wave order parameter. The other school refers to the growing body of empirical and theoretical evidence suggesting that the electrons have been eaten by dynamical stripes[1, 2]. The paradox is that stripes and nodal fermions are mutually exclusive.

The basic assumption underlying stripes is that the electrons are expelled from the magnetic domains, with the unavoidable consequence that the soft charge de-

grees of freedom are associated with the motions inside the stripes. Since the stripes are oriented along the  $(1,0)/(0,1)$  lattice directions the low energy propagating electronic excitations should be found along the  $\Gamma - X/Y$  directions in the Brillouin zone. Along  $\Gamma - M$  the excitations have to traverse the insulating domains and this should cause a severe damping if not a complete gapping. Instead, photo-emission shows relatively sharp dispersive features along  $\Gamma - M$  which are quite like the nodal fermions of a d-wave superconductor. The messy fermions are found along the stripe directions.

It seems a widespread reflex to postulate a ‘two-fluid’ picture: stripes and nodal fermions reflect separate universes, both governed by their own laws, which are for whatever reason completely disconnected. One has no other choice within the confines of BCS theory and the present understanding of dynamical stripes. However, this is no more than admitting defeat in the face of the paradox.

We want to pose the following question: can it be that stripes and nodal fermions are two manifestations of an underlying unity while they appear as dissimilar because of flawed hidden assumptions in the theory?

The remainder of this text will be a modest attempt to make the mind susceptible to the possibility that a positive answer exists for this question. A tactics will be followed which is not dissimilar to the habits in string theory. An alternative theory is suggested, of a highly speculative kind while its consequences are far from clear because of severe technical difficulties. However, it has the special merit that it does not suffer from the paradox, thereby shedding some light on what can be wrong in the current understanding.

The burden is on the stripe side. One has to get out of the narrow interpretations of textbook BCS theory to appreciate the nodal fermions on a sufficiently general level, and this work has already been done by others – see the next section. Our speculation is that the cur-

rent way of viewing dynamical stripes is too classical. Instead, we assert that the superconducting- and stripe states are related by duality (section 3). Static stripes and superconductivity are competing orders and the duality principle of quantum field theory states that the competing phases can be viewed as ‘two sides of the same coin’[3]: the disordered state (the superconductor) can be viewed as a condensate of the topological excitations (disorder fields) associated with the ordered state (the static stripes). The topological excitations of the stripe phase (‘stripe dislocations’) have such an unusual structure that it is a-priori unreasonable to assume that the associated disorder field-theory does not support nodal fermions.

### THE NODAL FERMIONS AS DIRAC SPINONS.

All what is needed on this side of the coin is to obtain a sufficiently general view on the nature of what has to be demonstrated: the nodal fermions. Although controversial[4], we will take here the conservative position that BCS is correct as the fixed point theory. One has to be, however, aware of the over-interpretations associated with the weak-coupling treatments in the textbooks. These are twofold: the ultraviolet is not governed by a non-interacting electron gas, even not to some degree of approximation[5]. Secondly, Bogoliubov quasiparticles are in fact  $S = 1/2$  excitations of the spin system (spinons) which acquire fully automatically a finite electron pole-strength in the superconducting state[6].

$H_0$  is not a Sommerfeld gas. The universality principle states that systems differing greatly at microscopic scales can nevertheless exhibit the same physics at macroscopic scales. BCS is a universal theory and its infrared structure can be deduced from a simple model. The standard textbook approach sets off by guessing a zero-th order Hamiltonian ( $H_0$ ) which depicts the large energy scale physics. In systems such as Aluminum, the Fermi-liquid renormalizations are basically complete at  $T_c$  and  $H_0$  is simply the Sommerfeld gas Hamiltonian. All one has to do is to add a small perturbation (the BCS-attractive interaction) which leaves the UV physics unaltered (the Fermi surface) while veering the system to the correct IR fixed point. Although the fixed point might still be the same, the way one gets there is entirely different in cuprates[5, 7]. There is no such thing as a close approach to the Fermi-liquids at short scales- and times as is the case in Al. Instead, the analysis of Shen and coworkers of the photo-emission suggests that at truly large energies the electrons move in stripes: the ‘holy cross’[8, 9]. Upon descending in energy, the cross starts to deteriorate and the nodal fermions start to appear. It is as if the nodal fermions are a long wavelength phenomenon associated with the quantum disordering of stripes!

It is only at low temperatures, deep in the supercon-

ducting state that one finds features which behave like quantum-mechanically propagating particles (the ‘quantum protection principle’[7]). The ramification is that it is not necessary to deduce a large, noninteracting Fermi-surface from quantum stripes. It is only necessary to demonstrate that the vacuum structure supports massless electron-like excitations living on Dirac cones: the nodal fermions.

*Nodal fermions are spinons.* As a next step, it is even not necessary to reinvent the electron. All that needs to be done is to find excitations carrying spin quantum number  $S = 1/2$  living on the Dirac cones. The superconducting condensate will take care of connecting these to the electrons. For this purpose we only have to remind the reader of an insight by Kivelson and Rokhsar[6], further elaborated by Fisher and coworkers[4]. According to the textbooks, the Bogoliubov quasiparticle is an electron because it has a pole-strength proportional to the square of a coherence factor. The finiteness of the pole-strength implies that the quantum numbers carried by the external electron can be attached to the excitations supported by the vacuum structure of the superconductor. Although spin- and momentum quantum numbers are sharply defined in the BCS state, there is a subtlety associated with the quantum of electrical charge: charge density is a fluctuating quantity in the superconductor and charge quanta can be added and removed at will from the condensate. Hence, the charge of the external electron can always be ‘dumped’ in the condensate.

To summarize, instead of reinventing aluminum, all what has to be done is to find out if a quantum disordered stripe phase can be constructed, which is superconducting while it carries  $S = 1/2$  excitations with a nodal-fermion dispersion.

### STRIPE DUALITY.

One of the quiet revolutions of mathematical physics is the discovery of the field-theoretic principle of duality. At first it appears as a mathematically rigorous procedure which can be carried through to the end in only a few simple cases (e.g., ref. [3]). However, it seems to reflect a physical principle of a far greater generality. Especially in the condensed matter context it has a stunning consequence: except for the critical state, the universality of duality seems to suggest that there are no truly disordered states at zero temperature. What appears as disorder is actually order of the disorder operators.

Duality can be formulated as an algorithm, with the following subroutines: (a) Characterize the order in the system in terms of an order parameter structure. (b) Enumerate the topological excitations, and link them to singular configurations of the order fields defined in (a). A single topological excitation suffices to destroy the order everywhere. (c) At a critical value of the coupling

constant these topological excitations will proliferate, signalling the transition to the ‘disordered’ state. (d) The constituents of the disordered state are the topological excitations of its ‘ordered’ partner. As these objects interact this in turn defines a ‘disorder’ field theory describing the condensation of the disorder matter. The ‘disordered’ state corresponds with an ordered state in terms of the topological excitations of the ‘ordered’ state.

Why should this have anything to do with the cuprates? Static stripes and superconductivity are clearly competing forms of order. When stripe order sets in, superconductivity is suppressed and vice versa. Moreover, it appears that this competition is governed by a (near) continuous quantum phase transition[2]. This is not unimportant, since duality is only rigorously defined in continuum field theory and therefore the characteristic length scales should be large as compared to the lattice constant. Dynamical stripes seem to fulfill this condition at least in the underdoped regime. Finally, the ordered stripe phase and the superconductor appear to be very different states of matter, but this is not an a-priori problem. After all, the central notion of duality is that one is supposed to be the ‘maximally disordered’ version of the other, although at elevated energies they are bound to merge in a single critical regime. The remainder of this section is intended to illustrate the problems encountered in the duality construction which are so severe that it cannot be excluded that it is actually the correct way of viewing these matters.

According to the duality recipe, we have to start out specifying precisely what stripe order means. A stripe phase is a highly organized entity and characterized by a variety of distinct, coexisting orders:

- (i) *The stripe phase is a Wigner-crystal.* This is obvious: the electrons form a crystal, breaking translational and rotational symmetry. We will adopt here the viewpoint that a fully ordered stripe state exists which can be used as reference state where translational symmetry is broken both parallel- and perpendicular to the stripes.
- (ii) *The stripe phase is a Mott-insulator.* We use here ‘Mott-insulator’ in the general sense that the charge order discussed under (i) is commensurate with the underlying crystal structure[9], causing a full gap in the charge excitation spectrum. This is actually controversial, and not of central importance in the present context. It is merely helpful, because there is nothing mysterious about an insulating stripe phase. Specifically, we will associate a conserved charge of  $2e$  to the stripe Mott-insulator, since with this choice the correct superconductivity emerges directly (see, however, [4]). The insulator would then correspond with a  $2k_F$  on-stripe density wave[10].
- (iii) *The stripe phase is a collinear antiferromagnet.* This is also obvious. Even when the charge order stays complete, the antiferromagnet can quantum disorder all by itself[11], and this is especially worth a consideration

in the bilayer systems. However, we will ignore this possible complication since the focus here is in first instance on the 214 system where the charge ordered systems seem always to be Néel ordered as well.

(iv) *The stripe phase is ‘topologically’ ordered.* This is the novelty of the stripe phase: whenever stripes are observed in cuprates and nickelates the charges are localized on the antiphase domain walls in the Néel state. It is intuitively clear that this is a form of order, although of an unusual kind. In the fully disordered stripe phase this ‘anti-phase boundariness’ must also be destroyed. Hence, the topological excitations of the topological order have to be considered and these are predominantly responsible for the unusual nature of the disorder theory.

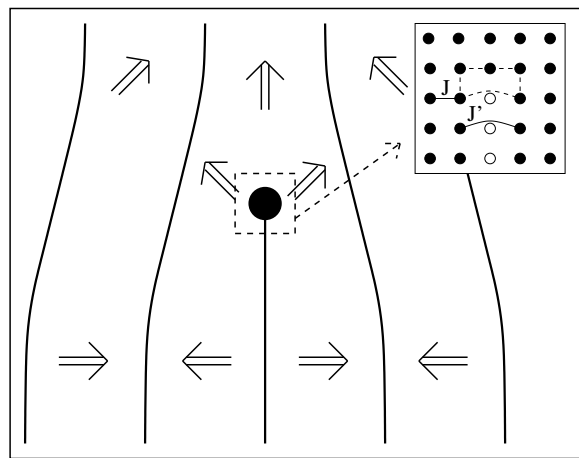


FIG. 1: Sketch of the stripe dislocation. The lines indicate the stripes and the arrows the direction of the Néel order parameter in the vicinity of the dislocation in the classical limit (‘ $\pi$ -vortex’.) The geometry of the ‘curved’ internal space seen by the spin system can be inferred from the exchange bonds indicated in the inset.

Given the complexity of stripe order, one anticipates a rather rich disorder-field theory. This is indeed the case. However, this structure can be built up starting from an elementary topological texture of a remarkable simplicity: the stripe dislocation as sketched in fig. 1. This is a stripe which is just ending somewhere in the middle of the sample. In the present context one should appreciate this object as a quantum particle, which can freely propagate through the lattice, occurring at a finite density in the quantum disordered stripe state.

The disorder fields are responsible for the fixed point physics in the disordered state, and these reflect the topological charges associated with the constituent topological excitations. What are the conserved charges associated with the stripe dislocations? Everything one needs to know for the charge sector is available, and the problems are associated with the topological- and spin sectors.

Let us first shortly discuss the charge sector – this will

be discussed in detail elsewhere. The ordered reference state is assumed to be a Mott-insulator, characterized by local conservation of charge. The stripe dislocation destroys this local charge conservation and is thereby an electrically charge particle carrying the charge quantum of the insulator. Assuming this charge to be  $2e$  and neglecting of the sign structure associated with the spin sector, the dislocations become hard-core bosons. Moreover, if the dislocations can move freely, then the infinitely long dislocation world lines of the dislocated state will wind around each other. The resulting entangled state is none other than a superconductor. This is just the inverse of the well known Abelian Higg's duality in 2+1 D[3]. This is not all, because the stripe phase is not just a featureless Mott-insulator but its charge sector also breaks translational- and rotational symmetry. The dislocation of fig. 1 is the topological defect associated with the restoration of translational symmetry, carrying a Burger's vector topological charge. Rotational symmetry is restored by a distinct topological excitation, and it is expected that these disclinations are initially suppressed. Although dislocations restore translational invariance, they leave the rotational symmetry breaking unaffected and this is the quantum-nematic state as introduced by Kivelson *et. al.* [12] (see also Balents and Nelson[13]). As will be discussed elsewhere [14], instead of the single nematic of Kivelson *et. al.* one finds actually a variety of physically distinct nematic like phases if one starts from a Mott-insulating stripe phase. For the present purposes all what matters is, however, that a state exists which is dislocated while the dislocations are subjected to 2+1D motions.

The stage is now set for the case we wish to make. The question is: what is the meaning of 'topological order' (or 'antiphase-boundarieness') and what does it mean to destroy this topological order? Our assertion is that the low energy effective theory associated with this 'order' is actually not an order parameter theory, but instead a *geometrical* theory. The spin system lives in a 'internal' space which is different from the space experienced by an external observer. In the absence of stripe dislocations this internal space is 'flat', but the dislocations are sources of 'curvature'. For the quantum antiferromagnet all that matters is the bipartiteness of the underlying lattice geometry. 'Curved' means that this bipartiteness is destroyed by the stripe dislocations. The spin system of the stripe-dual lives on a frustrating lattice which itself is fluctuating.

This can be discussed in a fairly rigorous setting, but given the space limitations let us just illustrate the main steps on an intuitive-geometrical level. What does antiphase boundarieness mean? In fact, it does not make sense to call stripes domain walls in the spin system. Domain walls occur when a  $Z_2$  symmetry is in charge and the (semiclassical) spin system is  $O(3)$  invariant. Stripes are in this sense non-topological and some other prin-

ciple is in charge, and this should be made explicit in order to construct the duality. This is a geometric principle and our claim is based on an exact result in 1+1D physics where 'antiphase boundarieness' is called 'spin-charge separation'.

Spin-charge separation has been demystified in a seminal contribution by Ogata and Shiba [15]. By inspecting the Bethe-Ansatz wave-function of the Hubbard model in 1+1D in the large  $U$  limit they come up with a particular prescription for constructing the spin dynamics. Although it does not seem to be fully appreciated, this involves the notions of a geometric theory: on the most basic level it is similar to the Einstein theory of gravity. Their prescription is as follows: choose a particular distribution of holes on the lattice, and the amplitude of this configuration in the wave function will be entirely given by the configuration of the charges. Every given distribution of holes defines a pure spin problem which is indistinguishable at large  $U$  from the Heisenberg spin chain *after a redefinition of the lattice*. This is the 'squeezing' operation: take out the holes, together with the sites where they reside, and substitute an antiferromagnetic exchange bond between the spins neighbouring the hole for the taken out hole+site. In a geometrical language, the external observer (us) experiences the full chain. However, the internal observer (spin system) experiences a different space: the squeezed chain where the holes and their corresponding sites have been removed. Although the internal observer is 'blind' for the charge dynamics, it does matter for the external observer and this gives rise to a particular simple factorizable form for the spin-spin correlation function measured by the latter. Since this correlator is universal, the geometric structure which it reflects is also universal, and apparently even realized in the weak coupling (Tomonaga-Luttinger) limit.

Taking this geometrical principle as physical law, how does it generalize to a higher dimension? The only feature of the embedding space which matters for the quantum antiferromagnet is the bipartiteness of the lattice. There are two ways of dividing a bipartite lattice in two sublattices and this defines a sublattice parity  $p$ :  $p = +1$  or  $p = -1$  if the covering is  $\dots A - B - A - B - \dots$  or  $\dots B - A - B - A - \dots$ , respectively. Divide now both the original- and the squeezed lattice in two sublattices: it is immediately seen that relative to the squeezed lattice the sublattice parity on the original lattice flips every time a hole is passed. The sublattice parity is the 'hidden'  $Z_2$  symmetry!

This generalizes in a unique way to the  $D$  dimensional bipartite lattice. In order to keep the bipartiteness intact in the absence of the holes, the holes have to lie on  $D-1$  dimensional manifolds. Hence, in 2+1D the holes are localized on 1+1D manifolds: the stripes. These manifolds can be of arbitrary shape in principle: the stripe fluctuations. Since the spin system on the higher dimensional squeezed lattice is unfrustrated, it will show long-range

Néel order. When the hole manifolds order, this spin order will also become manifest. We claim that this prescription is consistent with all available experiments on stripes. We emphasize that we take here a phenomenological stand: the reason that this happens should be given by microscopic theory and this is far from settled. However, if the interest is in the long wavelength behavior one might as well pose the principle and take it for granted as long as it is consistent with the experiments.

In 2+1D the form of order described in the above can be destroyed in a way which is impossible in 1+1D: the stripe dislocation is the topological excitation of the ‘topological’ (sublattice parity) order in 2+1 D. Although it can be stated more precisely, it is already clear from Fig. 1: the sublattice parity order of the upper part of the figure cannot be matched with the lower part. More precisely, the space experienced by the spin system (the squeezed lattice) is no longer bipartite. This loss of bipartiteness, a frustration, is analogous to spatial curvature. In prior works [16], geometrically frustrated systems have been investigated on their own right- there frustration was incurred by the noncommuting nature of the generators of translation. In the present context, the frustration inherent in the loss of bipartiteness may be similarly reformulated in a geometrically precise manner [14]. Charge (stripe) dislocations destroy spin charge separation and act as gravitational sources for the spin texture.

The analogy with gravity becomes more literal in the classical limit. Consider  $S \rightarrow \infty$  and static dislocations. The Néel order parameter texture is as indicated with the arrows in Fig. 1. This can be called a ‘ $\pi$ -vortex’ (see also [17]), since it looks like ‘half’ the topological excitation of a  $O(2)$  system. However, the spin system is  $O(3)$  invariant and the soliton of the  $O(3)$  system in 2+1D is the skyrmion, corresponding with a texture where the plane in which the order parameter rotates in internal space depends on the direction one takes in the embedding space. The rotation in Fig. 1 is in a single plane (like a vortex) and therefore it does not carry a conserved topological spin charge. Also notice that it is distinguished from a  $O(2)$  vortex because it carries a zero-mode. Ascribe the rotation as indicated in the figure to the equator of the  $O(3)$  sphere. Keeping the order parameter fixed at left- and right infinity, degenerate configurations are obtained by canting the order parameter ‘above’ the dislocation in the direction of one of the poles.

Interestingly, the above is exactly reproduced by embedding a  $O(3)$  sigma model in a 2+1D space with a metric given by Einstein theory in the presence of a mass source of strength  $8Gm = 1$  ( $G$  is Newton’s constant and  $m$  the mass). In this limit, stripe dislocations act like the famous ‘conical singularities’ of 2+1D gravity [18]. Unfortunately, the stripe dislocations are not Lorentz invariant, otherwise the semiclassical theory of quantum stripes would reduce to an exercise in 2+1D quantum

gravity!

Although these textures are non-topological, they are clearly ‘disorder operators’ in the spin system and when the stripe dislocations are proliferated while their spin zero-modes are also disordered, they will destroy the Néel order completely, giving rise to a dynamical mass-gap. However, there is a next subtlety: even when  $2e$  is chosen for the electrical charge quantum the theory can no longer be bosonic when free stripe dislocations are present. In order to see this, we have to go back to the lattice geometry. Take the Ogata-Shiba prescription and remove the charge-stripes, substituting an anti-ferromagnetic bond for the lattice sites where the stripe reside. The lattice geometry as seen by the spins is as indicated in the inset of Fig. 1. At the dislocation a ‘pentagon’ plaquette is found and this is directly recognized as a spin frustration event causing minus signs which cannot be transformed away.

The ground state wave function of a nearest-neighbor Heisenberg spin system on a bipartite lattice is nodeless. This is easily seen as follows. Keep the spin operators on the  $A$  sublattice fixed and regauge the spin operators on the  $B$  sublattice according to  $S^z \rightarrow S^z$  and  $S^\pm \rightarrow -S^\pm$  which leaves the commutation relations unaffected. In the basis which is diagonal in the Ising term, all off-diagonal matrix elements become negative and this means that the ground state wave function only contains positive definite amplitudes. Repeating this on the squeezed spin lattice associated with the stripe dislocation, one finds a seam of positive bonds, starting at the dislocation and ending at infinity. The location of this seam is without physical meaning; it is easily checked that by repeatedly applying the gauge transformations [14, 19] the sign-string can be moved arbitrarily through the plane, and the locus of the string is therefore a gauge freedom. Elsewhere we will argue that the spin-system is also insensitive to the locus of the half-infinite stripe attached to the dislocation and this means that the stripe dislocation the stripe dislocation appears in the spin system as a quantum particle attached to infinity by the sign string.

In the presence of irreducible signs mathematical physics comes to a grinding halt, and we are not aware of a precedent for the above sign structure. All one can say in general is that deep in the semi-classical regime signs are not immediately detrimental. Studies of the  $J1 - J2$  model show that the Néel state is robust against a substantial degree of geometrical frustration while the (spin-Peierls) physics found at optimum frustration can be understood without referral to Marshall signs[20]. However, also in the semi-classical case one encounters a problem with the above, which now takes the shape of a Wess-Zumino-Witten type Berry phase[21]. In the derivation of the semi-classical theory using the spin-coherent state path integral formalism one encounters imaginary terms in the Euclidean action

which are proportional to the topological (Berry-) phase which takes care of the quantization of the microscopic spin. In a many-spin system it takes the form  $S_{WZW} = S \sum_{\vec{r}} \int_0^1 d\tau \int_0^\beta dt \vec{n}(\vec{r}, t, \tau) \cdot \partial_t \vec{n}(\vec{r}, t, \tau) \times \partial_\tau \vec{n}(\vec{r}, t, \tau)$  ( $t$  is imaginary time). In the 1+1D case and for large  $S$ , this reduces to  $2\pi S q$ , where the integer  $q$  is the Skyrmion number associated with the order parameter texture in space-time. For half-integer spin this leads to alternating signs in the quantum partition function and these are believed to be responsible for the collapse of the mass gap of the integer spin systems[22].

It was pointed out that in the large  $S$  limit these topological terms are inconsequential for the 2+1D quantum antiferromagnet on the bipartite lattice[21]. This lattice can be divided into even and odd 1+1D rows, and the topological phase associated with the even ‘chains’ exactly compensate those of the odd ‘chains’. Consider now the stripe dislocation. Computing the topological phase for the ‘conical’ texture of Fig. 1, we find that the compensation is no longer complete. The texture can be smoothly deformed because the phase itself is topological, and it is easily demonstrated that it corresponds precisely with the 1+1D topological phase associated with the additional row in the lattice of half-infinite length, starting at the dislocation. Hence, even in the semi-classical case ‘sign’ problems remain although it is not at all clear to us what these imply.

### THE FAITH OF THE PARADOX.

What did we accomplish? In fact very little. Following the duality algorithm to the letter, we found that in combination with our understanding of the ‘antiphase-boundarieness’ of the stripes a novel problem is generated. We have no clue regarding the nature of the solution of this problem.

However, it is interesting to revisit the paradox discussed in the introduction. Its signature was that it was not possible to simultaneously take stripes and nodal fermions seriously. In this stripe-duality framework this is no longer true. The paradox has been resolved to yield a question: could it be that the stripe-disorder fields support nodal fermion excitations?

Let us first completely neglect the signs and in this case we know what to do. In the superconducting phase the world lines of the dislocations are winding around each other. To every world line a spin texture is attached of the kind as indicated in Fig. 1 – the signature of the spin system as it appears in the inelastic neutron scattering suggests that the spin system can be considered as semiclassical and since the spin-wave velocity is large it might well be that the spins can follow the charge motions nearly instantaneously. The ‘ $\pi$ -vortices’ are clearly disordering events in the spin system and, interestingly, they exert this disordering influence in the same way in

all the directions in space. The ‘ $\pi$ -vortex’ covers the half-infinite plane ‘above’ (fig. 1) the dislocation and since the dislocation occurs at all ‘vertical’ positions the spin system is disordered identically in all directions. A quantum fluid of ‘ $\pi$ -vortices’ does not know about the directionality of stripes. This is a somewhat too rigorous resolution of the stripes-nodal fermion paradox: a dynamical mass gap should be generated in the spin sector and this gap should be rather uniform in momentum space, because of the isotropic disordering influence of the  $\pi$ -vortices.

Fortunately, there are the minus signs. Although little can be said in general, they do cause destructive interferences and have a reputation to diminish spin-gaps in favor of massless spinon excitations. The effective spin problem to be solved is that of a quantum-antiferromagnet living on a bipartite lattice pierced by the local frustration events associated with the stripe dislocations which themselves are moving around quantum mechanically. Is there any reason to exclude that this behaves like a d-wave superconductor?

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